Mental Calculation Progression

Summer 2022

The National Curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils have conceptual understanding and are able to recall and apply their knowledge rapidly and accurately to problems
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

This document aims to support teachers with a map of progression in mental fluency. The programme of study includes references to mental calculation but lacks the detail needed to provide a coherent pathway. It is up to schools to decide upon what this should look like. This guidance document and links to the appendix provides the necessary detail.

Practice

Practice is a key approach to developing the automaticity needed to reduce cognitive load. Pupils who have facts and skills at their fingertips are more likely to attend to the particulars of new learning than those that do not. These pupils have to work harder and are overburdened. Practice not as meaningless repetition of facts in which pupils chant without thought or as a series of isolated facts learnt at home then tested in school, but as a chance to rehearse them within exercises that develop better thinking. Practice is an opportunity to keep facts and skills 'simmering' and a further chance to vary the ways that they are presented. Schools should be mindful of the *quality* of practice rather than the *quantity*. Similarly, they are advised to focus upon the facts and skills that will make the greatest difference to mental fluency at each phase. As a school we dedicate an additional fluency session to allow teachers to focus on the development of these key skills. When supporting children to develop their recall of number facts and to ensure we have a consistent approach across the school, we use:

- Number Sense Maths for addition and subtraction in KS1
- Calculation strategies from Mental Calculation Progression and Calculation strategies Y1 –

Y6.

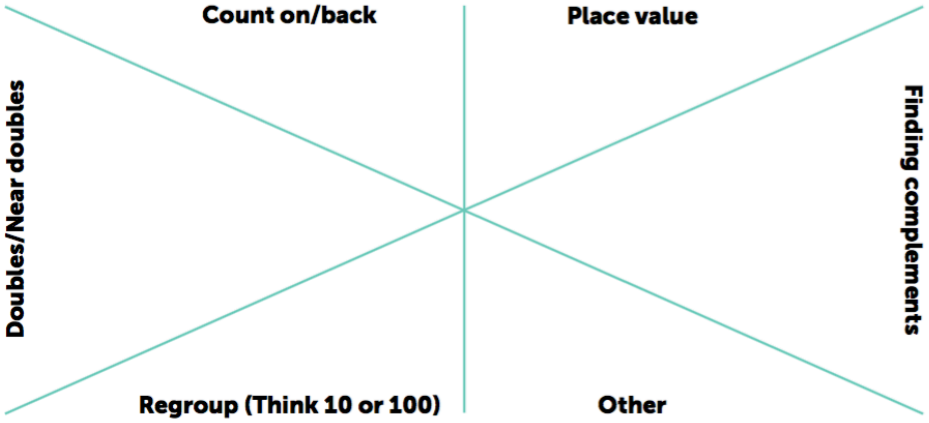
- Times Tables Challenges for multiplication and division and TT Rockstars.

Strategies to be taught and practised:

Subitising	the ability to see number as a pattern, such as dice patterns. This supports pupils to see numbers within numbers and better regrouping (partitioning).
Regrouping (partitioning)	the ability to break numbers up and recombine them flexibly. *Regrouping – can be categorised as partitioning and recombining e.g. splitting into tens and ones. Regrouping can also be used to describe bridging through the boundaries e.g. 7 + 5 – split the 5 into a 3 and a 2 to make the 7 into a 10.
Counting on and counting back	in a variety of interval steps.
Reordering	knowing when and how to reorder to make calculations easier.
Finding complements	links to reordering, identifying useful complements pairs or trios of 1, 10, 60 etc.
Applying the inverse	use of fact family knowledge to 'undo'.
Rounding	to a range of benchmark numbers.
Estimation	both linear estimation on number lines and scales, and of quantities and calculations to support an increasing sense of what is reasonable.
Compensation	to use rounding to add or subtract too much or too little and adjust accordingly. Compensation is also called round and adjust e.g. 24 + 9 is equal to 24 + 10 and then subtract 1.
Rebalancing	to adjust the parts of addition and subtraction facts to make a calculation easier. **Equal sum** for addition – 57 + 24 would be equal to 57 + 3 and then do 24 – 3. The new calculation becomes 60 + 21. **Same difference** for subtraction – 54 – 27. We add 3 to both numbers to keep the difference the same and to make the calculation easier by adjusting the amount you are subtracting into a friendly number (multiple of 10/100). 57 – 30.
x/÷ by powers of 10	

Doubling and halving	also useful for multiplication e.g. 18 x 5 becomes 9 x 10.
Rearranging	to adjust the groups in multiplication and division to make a calculation easier.

When solving a calculation children need to decide which strategy is most efficient. Ask the children – How could you solve this?



Key Stage One

Key facts	
Year One Recall • Number bonds within 10 including a + b + c = d, the effect of adding zero and missing number calculations • Reordering to find tens and some more e.g. 4 + 5 + 6 = • Doubles within 10 including subtraction e.g. 6 - 3 = 3 and missing numbers e.g. 6 - ☐ = 3 • Structured subitisation on tens frame to 20	**Year Two Recall** • Addition and subtraction facts to 20 • Multiplication and division facts 2, 5 and 10 x tables • Multiplication facts for 3 x tables • Number of minutes in an hour; number of hours in a day • Coin recognition up to £2 • Doubles to 20

Key Stage 1 Examples

'Think 10' Regroup
5 + 6 7 + 4 9 + 7 7 + 6 8 + 7 7 + 5
2 + 18 4 + 18 8 + 19 47 + 6 68 + 7 9 + 87
13 - 8 27 - 8 53 - 6 68 - ☐ = 7 73 + ☐ = 89

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper
8 + 6 = ☐ 5 + 7 = ☐ 12 - 7 = ☐ 46 + 7 = ☐
8 + 5 + 4 = ☐ 55 + 17 = ☐ 71 - 14 = ☐
86 - 21 = ☐ 65 + ☐ = 93

'Think Addition' for subtraction
8 - 5 9 - 6 6 - 2 80 - 50 19 - 6 60 - 20

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper
10 - ☐ = 2 ☐ + 5 = 9 12 - 7 = ☐ 19 - 9 = ☐
17 - 6 = ☐ 39 - 8 = ☐ 50 - ☐ = 20 56 - ☐ = 51

Reordering and finding complements
5 + 4 + 5 2 + 3 + 8 2 + 4 + 6 6 + 3 + 7
36 + 5 + 4 54 + 26

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper
4 + 5 + 6 = ☐ 36 + 24 = ☐ 69 + 11 = ☐

Compensation
2 + 9 12 + 9 9 + 72 2 + 19 19 + 42 42 + 39
5 + 8 15 + 8 65 + 8 18 + 5 55 + 18 48 + 35
12 - 9 22 - 9 52 - 9 52 - 19 92 - 19 92 - 39
12 - 8 22 - 8 52 - 8 52 - 18 92 - 18 92 - 48
48 + ☐ = 92 8 + ☐ = 52

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper
8 + 6 = ☐ 52 + 7 = ☐ 28 + ☐ = 35 69 + 11 = ☐
55 + 17 = ☐ 39 - 8 = ☐ 43 + 38 = ☐ 70 - 18 = ☐

Rebalancing - Equal sum
12 + 9 9 + 72 24 + 19 15 + 42 44 + 37
5 + 8 15 + 8 65 + 7 18 + 6 55 + 15
48 + 35

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper
8 + 6 = ☐ 4 + 5 + 6 = ☐ 69 + 11 = ☐
55 + 17 = ☐ 36 + 24 = ☐ 43 + 38 = ☐
8 + 5 + 4 = ☐

Rebalancing - Equal difference
32 - 7 25 - 8 55 - 7 55 - 17 92 - 19
97 - 43 48 + ☐ = 92 8 + ☐ = 55

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper
12 - 7 = ☐ 28 + ☐ = 35 71 - 14 = ☐
39 - 8 = ☐ 86 - 21 = ☐ 70 - 18 = ☐
65 + ☐ = 93

Reordering and multi-strategy
Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper
8 + 5 + 4 = ☐

Double and near double facts
3 + 3 30 + 30 32 + 32 3 + 4 30 + 40
6 - 3 60 - 30 64 - 32 7 - 3 70 - 40
6 - ☐ = 3 60 - ☐☐ = 30 64 - ☐☐ =
☐☐ - 3 = 64

Find two ways of solving this: 70 - ☐0 = ☐0

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper
12 - 7 = ☐ ☐ + 5 = 9 50 - ☐ = 20

Make links to doubling and halving
3 + 3 = 2 x 3 2 x 30 2 x 3 + 1
6 ÷ 2 60 ÷ 2

Ensure that pupils can halve odd multiples of ten
50 ÷ 2 =

Examples from 2016 KS1 Paper 1 and Sample Arithmetic Paper
3 x 2 = ☐ 2 x 0 = ☐
8 ÷ 2 = ☐ ½ of 16 = ☐ ½ of 30 = ☐
12 ÷ 2 = ☐

Lower Key Stage Two

Key facts	
Year Three Recall • Sums and differences between pairs of numbers which are multiples of 10 and 100 • Doubles and halves of multiples of 10 or 100 • Complements to 100 • Complements to 60 (time) • Complements of tenths that make 1 • Complements of fractions with the same denominator that make 1 e.g. 3/7 + 4/7 = 1 • x 3, x 4, x 8 facts including division facts • Number of seconds in a minute • Number of days in a month and in a year including a leap year	**Year Four Recall** • Review addition and subtraction facts within 20, ensure application to 10, 100 and 1000 (6 + 3, 60 + 30, 600 + 300, 6000 + 3000) • Doubles and halves of multiples of 10, 100 or 1000 (6 + 6, 60 + 60, 600 + 600, 6000 + 6000) • All multiplication and division facts to 12 x 12 • Multiplication and division by zero and one facts • Division and multiplication by 10 and 100 • Conversion of kilometres to metres, hours to minutes, years to months, weeks to days • Complements of hundredths that make 1

Lower KS2 examples

'Think Regroup' for addition

Think 10
37 + 45 68 + 23 29 + 75
76 + 27 55 + 16 42 + 38
537 + 8 727 + 5 213 + 18 146 + 37
36 - □ = 29 56 - 2□ = 33 □7 – 45 = 32

Think 100
290 + 13 370 + 50 580 + 73
270 + 51 67 + 350 860 + 69
86 + 770 680 + 63

Think 1000
4900 + 500 4800 + 260 6900 + 430
3200 + 910 230 + 7900 570 + 8500
3700 + 370 3622 + 500

Think 1
2.7 + 1.4 $2\frac{8}{10} + \frac{3}{10} =$ 6.5 + 5.6
$1\frac{7}{8} + 1\frac{5}{8}$

'Think Regroup' for subtraction

Think 10
97 - 8 74 - 7 53 - 5 63 - 37
77 - 32 84 - 26 57 - 28 256 - 37
25 + □= 85 163 + □ = 363 426 + 2□2 = 668

Think 100
230 - 70 660 - 82 420 - 77 950 - 147

Think 1
1.3 - 0.6 $1\frac{4}{8} - 1\frac{5}{8}$ 3.4 - 2.7 $2\frac{1}{3} - 1\frac{2}{3}$

Re-ordering and finding complements

Complements to 10
8 + 6 + 2 + 3 + 4 3 + 5 + 7 + 5 +4
1 + 4 + 6 +7 + 9 30 + 50 + 70
25 + 50 + 5 75 + 40 + 20 + 25

Complements to 100
400 + 547 + 600 700 + 240 + 300
750 + 400 + 250

Complements to 1
2.7 + 4 + 1.3 4.6 + 5 + 2.4 8.2 + 3 + 5.8

Compensation
42 + 29 45 + 27 24 + 47 28 + 65 68 + 27
232 + 49 856 + 17 48 + 325 232 + 95
132 + 59 568 + 195 399 + 423 412 + 298
405 + 199 597 + 308
43 - 18 94 - 37 54 - 29 77 - 9 82 - 23
483 - 99 256 - 98 398 - 74 597 - 63
401 - 97 736 - 301 613 - 299 743 - 397
298 - 156 799 - 403

Rebalancing - Equal sum
45 + 27 26 + 39 78 + 18 65 + 27
73 + 39 84 + 47 42 + 97 116 + 35
368 + 123 404 + 198 356 + 427 528 +298
3.7 + 1.9 7.6 + 4.7 1.9 + 5.8

Rebalancing - Equal difference
75 - 28 56 - 29 78 - 38 55 - 27
83 - 21 75 - 12 95 - 42 67 - 51
912 - 797 837 - 498 711 - 467 628 -198
482 - 302 729 - 404 548 - 202 637- 203
6.4 - 3.9 6.6 - 3.2 7.7 - 4.8 $1\frac{2}{7} - \frac{5}{7}$

Counting on to subtract
315 - 298 412 - 396 917 - 898 611 - 598

Think multiplication
85 ÷ 5 72 ÷ 4 99 ÷ 6 240 ÷ 12
660 ÷ 3 210 ÷ 7 540 ÷ 9 500 ÷ 4
□ ÷ 3 = 8 3□ ÷ 5 = 6

X and ÷ 10, 100 and 1000
4 x 30 9 x 30 70 x 70 60 x 50
300 x 4 800 x 7 9 x 800 6 x 400
3 x 2000 4000 x 6 8 x 7000 9 x 8000
500 ÷ 10 400 ÷ 5 600 ÷ 2 240 ÷ 4
120 ÷ □ = 12 365cm = □ m 750mm = □ cm

Double and near double facts
7 x 20 3 x 38 9 x 200 11 x 4
16 x 20 18 x 2000
80 ÷ 4 160 ÷ 4 1600 ÷ 4 2400 ÷4

Think 5 / Think 10 for multiplication
28 x 5 16 x 8 23 x 9 92 x 8 52 x 4
13 x 21 34 x 19 123 x 4 214 x 6 9 x 234
11 x 314 21 x 400 400 x 38

Upper Key Stage Two

Upper KS2 examples

Place Value

937 + 100 1969 + 100 546 - 40
1.7 + 0.05 40 000 - 500
246 ÷ 1 100 x 217 0.4 ÷ 10
1.68 x 100 100 x 100

Examples from 2016 KS2 and Sample Papers

435 - 30 979 + 100 3.005 + 6.12 2.15 + 0.05
100 x 412 0.9 ÷ 10 1.28 x 100 50,000 – 500
10 x 100

Two decimal numbers add together to equal 1 One of the numbers is 0.007. What is the other number?

Circle two numbers that added together make 0.25
0.05 0.23 0.2 0.5

Circle two numbers that multiply together to equal 1 million 200 2,000 5,000 50,000

Write the number that is 5 less than 10 million

Write the number that is one hundred thousand less than six million

Round 124,531 to the nearest 10,000, 1,000, 100

Think Regroup

58 + 6 5 + 47 630 + 73 680 + 78
560 + 89 8900 + 230
74 - 7 97 - 8 320 - 50 2300 - 600
3400 - 1700
5 - 2.65 8.1 - 2.75 $1\frac{2}{5}+\frac{3}{10}=$ $1\frac{3}{10}-\frac{2}{5}=$
£3367.40 - £1021.23

Examples from 2016 KS2 and Sample Papers

4 - 1.15 $1\frac{4}{5}+\frac{3}{10}$ $1\frac{1}{4}+\frac{1}{3}$ $1\frac{1}{5}-\frac{1}{4}$ $\frac{3}{4}+\frac{7}{8}=$
5,756 + 8,643 936 + 285

Compensation

56 + 8 72 + 9 56 - 8 72 - 9
371 + 18 255 + 49 304 + 299
673 - 99 854 - 398 3720 - 996
0.71 + 0.09 0.56 + 0.08 0.34 - 0.09
£1.17 + £0.39 £8.89 - £4.99

Examples from 2016 KS2 and Sample Papers

468 - 9 472 - 9 15.98 + 26.314
12 - 6.01 15.4 - 8.88

Rebalancing - Equal sum

56 + 8 72 + 9 371 + 18 255 + 49
304 + 267
£37.67 + £3.85 563 + 397 890,488 + 4,890
229,899 + 31,321

Examples from 2016 KS2 and Sample Papers

89,994 + 7,643 936 + 285 89,994 + 7,643

Rebalancing - Equal difference

85 - 18 42 - 17 88 - 43 437 - 103
819 - 504 532,525 - 9897
£122.56 - £87.99 9.1 - 6.7 15.3 - 5.7

Examples from 2016 KS2 and Sample Papers

468 - 9 472 - 9 122,456 - 11,999
4 - 1.15 12 - 6.01
15.4 - 8.88 234,897 - 45,996

Think Partition for x and ÷

32 x 4 29 x 2 122 x 4 4.6 x 2
75 x 3 8.3 x 6 39 x 7
3.3 x 7 5 x 49 4 x 198 96 x 0.3

Examples from 2016 KS2 and Sample Papers

15 x 6.1 24 x 3 1.52 x 6 7,505 ÷ 5
17 x 1½

Make links to doubling and halving

50 x 28 86 x 50 500 x 70 18 x 2.5
86 x 2.5 160 x 35 500 x 88 1.5 x 6.6
0.5 x 120 4.5 x 2.2 15% x 346 75% x 220

Examples from 2016 KS2 and Sample Papers

15% x 440 $\frac{2}{5}$ x 140 24 x 3
20% of 1500 95% of 240

Multiplying and dividing fractions

Examples from 2016 KS2 and Sample Papers

$\frac{3}{5}\div 3$ $\frac{2}{5}\div 2$ $\frac{3}{4}\div 2$ $\frac{2}{5}\times 140$ $\frac{1}{4}\times\frac{1}{8}$

Re-ordering and finding complements

11 + 59 33 + 57 14 + 90 + 86
290 + 310 1.15 + 2.55 0.8 + 0.26

Examples from 2016 KS2 and Sample Papers

1,034 + 586 2.15 + 0.05

Circle two numbers that added together make 0.25
0.05 0.23 0.2 0.5

x and ÷ by powers of 10

10 x 53 87 x 10 1000 x 14 100 x 8.3
100 x 0.41
30 x 3 7 x 0.3 30 x 30 30 x 70
567 ÷ 100 36 ÷ 10 0.5 ÷ 10 280 ÷ 4

5600 ÷ 80 30 = □ ÷ 12 270 ÷ 9 = □ ÷ 0.9
7 x 0.001 1.8 ÷ 0.1 3.25 ÷ 0.00001

Circle two numbers that multiply together to equal 10 million
200 2,000 5,000 50,000

Examples from 2016 KS2 and Sample Papers

1440 ÷ 12 630 ÷ 9 1,320 ÷ 12
0.9 ÷ 10
20% of 1,800 20% of 1500 7,505 ÷ 5
95% of 240
100 x 412 0.9 ÷ 10 1.28 x 100
50,000 - 500 10 x 100

Circle two numbers that multiply together to equal 1 million
200 2,000 5,000 50,000

Appendix

See calculation strategies with visual representations for each number operation and each year group:

Q drive – Maths – Mental calculation policy